$N = 1$ SUSY Yang-Mills Theory on the Lattice

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Introduction

Lattice simulations allow the systematic investigation of strongly interacting SUSY gauge theories in the IR regime. Our studies are focused on the $\mathcal{N} = 1$ Super Yang-Mills theory with gauge group SU(2), described by the action

$$S_{SYM} = \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} \bar{\lambda}^a \gamma^\mu D_\mu \lambda^a \right\}.$$

The aim of our investigations is to determine the mass spectrum of the low-lying bound states of the theory, which is expected to form two supermultiplets consisting of gluinoballs, glueballs and gluino-glueballs. By using new algorithms, we are able to determine the spectrum in large volumes. In our presently running simulations are performed on $16^3 \times 32$ and $24^3 \times 48$ lattice at $\beta = 1.6$ corresponding to a box size $L \geq 2$fm.

Lattice action

We use the Wilson discretization of the action. To lower artifacts, induced by the finite lattice spacing, which breaks supersymmetry and chirality explicitly, we have implemented the tree-level improved Symanzik action for the gauge part, and we use one step of STOUT-smearing (smearing parameter $\rho = 0.15$) for the fermion part. This is advantageous in view of a speed up of the simulations and has beneficial effects in the analysis. [Jansen, 2007]

$$S = \underset{\downarrow}{S_g} + \underset{\downarrow}{S_f}$$
$$\text{tlSym} \quad \text{STOUT}$$

The gauge action with tree-level improved Symanzik action has the form

$$S_g = \beta \left(c_1 \sum_{plaq} \mathrm{ReTr} \left(1 - \frac{1}{3} U_{plaq} \right) + c_2 \sum_{rect} \mathrm{ReTr} \left(1 - \frac{1}{3} U_{rect} \right) \right)$$

$U_{plaq} =$  $\qquad U_{rect} =$ 

The fermionic action reads

$$S_f \left[U, \bar{\lambda}, \lambda \right] = \frac{1}{2} \sum_{xy} \lambda^T (x) \mathcal{C} Q_{x,y} \lambda (y)$$

in which the Q-Matrix

$$Q_{y,x}[U] \equiv \delta_{yx} - \kappa \sum_\mu \left[\delta_{y,x+\hat{\mu}} \left(1 + \gamma_\mu \right) V_\mu (x) + \delta_{y+\hat{\mu}} \left(1 - \gamma_\mu \right) V^T_\mu (y) \right]$$

and the adoint matrices

$$\left[V_\mu (x) \right]_{ab} \equiv 2\mathrm{Tr} \left[U^\dagger_\mu (x) T^a U_\mu (x) T^b \right]$$

are defined.
In the fermionic matrix, we use STOUT link smearing. The $(n+1)^{th}$ stout smeared "thick" link obtained iteratively from the n^{th} level

$$U^{(n+1)}_\mu (x) = \mathrm{e}^{iQ^{(n)}_\mu} U^{(n)}_\mu (x)$$

with

$$Q_\mu (x) = \frac{i}{2} \left(\Omega^\dagger_\mu (x) - \Omega_\mu (x) \right) - \frac{i}{2N} \mathrm{Tr} \left(\Omega^\dagger_\mu (x) - \Omega_\mu (x) \right)$$

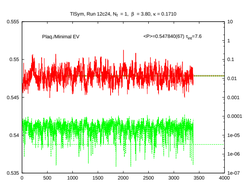
and

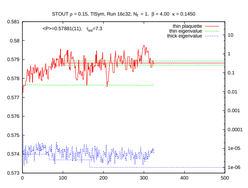
$$\Omega_\mu (x) = C_\mu (x) U^\dagger_\mu (x)$$

the staples C_μ are defined as

$$C_\mu (x) = \sum_{\nu \neq \mu} \rho_{\mu\nu} \left(U_\nu (x) U_\mu (x + \hat{\nu}) U^\dagger_\nu (x + \hat{\mu}) + U^\dagger_\nu (x - \hat{\nu}) U_\mu (x - \hat{\nu}) U_\mu (x - \hat{\nu}) U_\nu (x - \hat{\nu} + \hat{\mu}) \right)$$

An important effect of these improvements is to rise the minimal eigenvalues and to give a further optimization of the condition number. In addition we expect a shorter autocorrelation time.





Aim of investigations

1. Our main goal is to determine the spectrum of bound states in large volumes with $L \geq 2$fm This will be a substantial improvement of the previous simulations by this collaboration on small volumes with lattice extensions $L \approx 1$fm. In our presently running simulations on $16^3 \times 32$ lattice at $\beta = 1.6$ the lattice extensions are about $L \equiv 2$fm. This we want to supplement by simulations on $24^3 \times 48$ lattice ($L \equiv 3$fm) and $32^3 \times 64$ lattice ($L \equiv 4$fm). In such large volumes the finite volume effects are most probably negligible. We can check this by comparing the results on three increasing volumes.
2. We are going to approach the continuum limit by performing simulations at $\beta = 1.8$ on $24^3 \times 48$ lattices.
3. We plan to investigate the first order chiral phase transition at zero gluino mass on larger lattices compared to our previous study on $6^3 \times 12$ lattice. [Kirchner et.al. 1999]

Update algorithm

The numerical simulations are done with an update algorithm, which is based on a multi-level polynomial approximation of the quark determinant and on stochastic update corrections. It is a development and combination of the polynomial hybrid Monte Carlo algorithm (PHMC) and the two step multi-bosonic algorithm (TSMB) [Montvay, Scholz, 2005]. We therefore integrate out the fermion fields and so obtain the square root of the fermion determinant

$$\int [d\lambda] \, e^{-S_f} = \int [d\lambda] \, e^{-\frac{1}{2} \bar{\lambda} Q \lambda} = \pm \sqrt{\det Q}.$$

The fermion measure can be derived by "bosonification". After a polynomial approximation

$$\lim_{n \to \infty} P_n (x) = [1/x]^{\frac{1}{4}} \qquad \forall x \in [\epsilon, \lambda]$$

we get with $\sqrt{\det Q} = \left[\det \left(Q^\dagger Q \right) \right]^{\frac{1}{4}}$

$$\left[\det \left(Q^\dagger Q \right) \right]^{\frac{1}{4}} = \int \left[d\phi^\dagger d\phi \right] \exp \left(- \sum_{xy} \phi^\dagger_y \left(P_n \left(\tilde{Q}^2 \right) \right)_{yx} \phi_x \right).$$

If necessary, we evaluate reweighting corrections including the sign of the fermion determinant.
The fields are updated by the polynomial hybrid Monte-Carlo algorithm. The Hamiltonian

$$H[P, U, \phi] = \frac{1}{2} \sum_{x\mu j} P^2_{x\mu j} + S_g [U] + S_f [U, \phi]$$

together with the equations of motion,

$$U'_{x\mu} = \exp \{ \sum_j i 2 T_j P_{x\mu j} \Delta\tau \} U_{x\mu},$$
$$P'_{x\mu j} = P_{x\mu j} - D_{x\mu j} S [U, \phi] \Delta\tau.$$

define the updates of fields and momenta. Here you have to calculate the derivation of the fermionic action,

$$\left[D_{x\mu j} V_\mu \right]_{ab} = 2 f_{bjc} \left[V_\mu \right]_{ac}.$$

Improvements

The different parts of the update process are sticked together according to the higher order Sexton-Weingarten integrator with multiple timescales.
To lower the polynomial order and therefore speed up the system, we implemented the two-step polynomial algorithm

$$1/x^{\frac{1}{4}} \equiv P_{n_1,n_2} (x) = P'_{n_1} (x) P''_{n_2} (x)$$

with a noisy correction. To obtain lower conditioned fermion matrices, the even-odd preconditioning is used.

$$\tilde{Q} = \begin{pmatrix} \gamma_5 & -\gamma_5 \kappa M_{even-odd} \\ -\gamma_5 \kappa M_{odd-even} & \gamma_5 \end{pmatrix}$$
$$\rightarrow \det \tilde{Q} = \det \left(1 - \kappa^2 M_{oe} M_{eo} \right).$$

Another advantage of this optimization is, that the matrix order is halved.
In addition, a determinant breakup with $n_B = 2$ is used

$$\det \tilde{Q}^2 = \left\{ \left(\det \tilde{Q}^2 \right)^{\frac{1}{n_B}} \right\}^{n_B}.$$

Deflation

For the calculation of the disconnected parts of propagators, we use the "Stochastic Estimator Technique" (SET). This part of the calculation takes a major part of our CPU time in the analysis procedure, so we are working on deflating methods based on the ansatz of Stathopoulos and Orginos, to improve matrix inversions. [Orginos, Stathopoulos, 2007] Consider e.g. the gluino-propagator

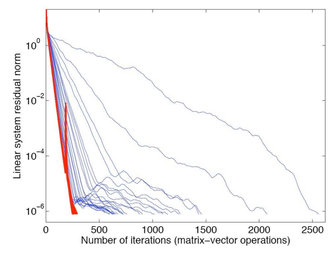
$$\left\langle T \left\{ \lambda (x) \bar{\lambda} (x) \right\} \right\rangle = \left\langle Q^{-1}_{xx} [U] \right\rangle.$$

With SET, we calculate the disconnected part with

$$\left\langle \eta^\dagger_i Z_i \right\rangle_{N_{est}} \overset{N_{est} \to \infty}{=} Q^{-1}_{ii}$$

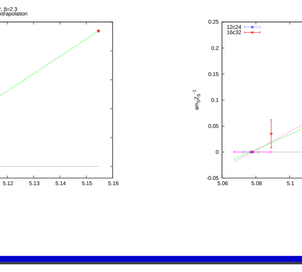
where η_i is a given source and Z_i is the solution vector of $Q_z = \eta$. To accelerate the matrix inversion the deflating algorithm of Stathopoulos and Orginos will be used further on, which is well suited to extend the Conjugate Gradient algorithm to get a faster convergence.

- In the first s steps, an eigCG algorithm is used. This is a Conjugate Gradient with restarting, in which matrix is collected, initially consisting of the residual vectors of the CG iteration step. The smallest eigenpairs are saved.
- The second and ongoing estimators are Galerkin-projected on a Krylov subspace $x_0 = W \left(W^T A W \right)^{-1} W^T b$ and then inverted by CG.
- In the $i \geq s$ steps, the estimators are Galerkin-projected and then they will be directly put in CG, because the Krylov subspace is approximated well enough and there is no need to search for the minimal eigenpairs. This part is called InitCG. The convergence of the algorithm increases after each iteration:



Analysis

We measure the low lying hadron spectrum, i. e. masses of $m_{a-\pi}$, m_{0^+}, m_{a-f_0}, $m_{a-\eta'}$ and $m_{\tilde{g}g}$.



κ	$am_{a-\pi}$	0^+(glub.)	a-f_0	a-η'	$\tilde{g}g_1$	$\tilde{g}g_{\gamma_0}$
0.194[†]	0.484(1)	$< 0.52(1)$	< 0.78	0.52(2)	0.40(7)	0.43(1)
0.1955	0.345(3)	$< 0.4(2)$	0.642(-)	0.48(1)	0.673(20)	0.700(21)
0.196	0.264(5)	$< 0.75(9)$	0.814(84)	0.399(45)	0.424(22)	0.472(31)
0.1965	0.208(13)	$< 0.28(3)$	-	0.44(14)	0.469(85)	0.472(75)

OZI arguments imply

$$(am_{a-\pi})^2 = A(\frac{1}{\kappa} - \frac{1}{\kappa_{cr}}).$$

Extrapolation: $\kappa^\pi_{cr} \sim 0.1969$
SUSY Ward-Identities: renormalized gluino mass

$$am_{\tilde{g}} Z_S = \frac{1}{2}(\frac{1}{\kappa} - \frac{1}{\kappa_{cr}}).$$

Extrapolation: $\kappa^{WI}_{cr} \sim 0.1969$