supersymmetric insights

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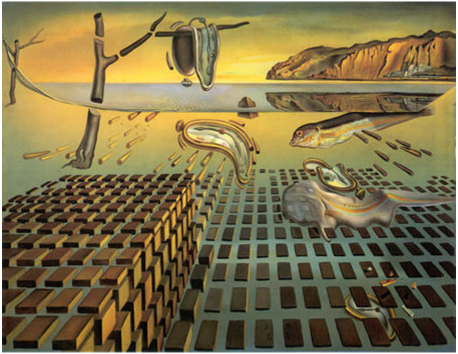


why supersymmetry?

- aim of investigations: constructing a theory with only one symmetry-group and delivering the standard model by symmetry breaking
- generators of the $\mathcal{P}$oincaré and Lie-Group $\mathcal{I}$:

$$[P^\mu, P^\nu] = 0$$

$$[M^{\mu\nu}, P^\rho] = i\,(\eta^{\nu\rho}P^\mu - \eta^{\mu\rho}P^\nu) \qquad \leftrightarrow \qquad [T_a, T_b] = f_{abc}T_c$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = i\,(\eta^{\nu\rho}M^{\mu\sigma} + \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\mu\rho}M^{\nu\sigma} - \eta^{\nu\sigma}M^{\mu\sigma})$$

- $\rightarrow$ non-trivial it is impossible
  (no-go theorem from Coleman & Mandula '67)

to crack this theorem

- extend the algebra by anti-commutating vectors instead of commutating ones GOLFAND & LIKHTMAN ('71)

$$\begin{aligned} [\text{even}, \text{even}] &= \text{even} \\ \{\text{odd}, \text{odd}\} &= \text{even} \\ [\text{even}, \text{odd}] &= \text{odd} \end{aligned}$$

- this so called $\mathbb{Z}_2$ graded algebra is the only one, consistent with relQFT

the majorana-spinors Q_α

- anti-commuting values are associated with fermionic degrees of freedom → generators are majorana/weyl-spinors.

self-conjugated complex/real dirac spinors

$$\psi_M^C = \psi_M$$

- with commutation- and anticommutation relations

$$\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$\{Q_\alpha, Q_\beta\} = \left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} = [Q_\alpha, P_\mu] = \left[\bar{Q}_{\dot\alpha}, P_\mu\right] = 0$$

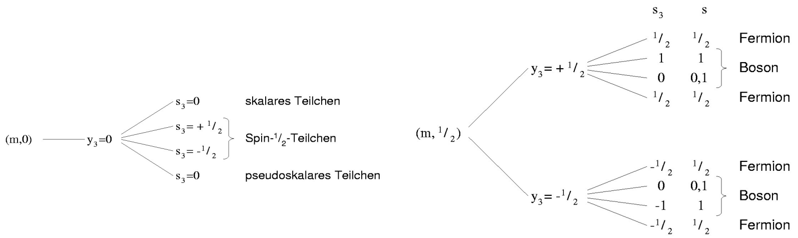
$$[Q_\alpha, M_{\mu\nu}] = \sigma^{\beta}_{\mu\nu\alpha} Q_\beta \qquad \left[\bar{Q}^{\dot\alpha}, M_{\mu\nu}\right] = \bar{\sigma}^{\dot\alpha}_{\mu\nu\dot\beta} Q^{\dot\beta}$$

the particle spectrum

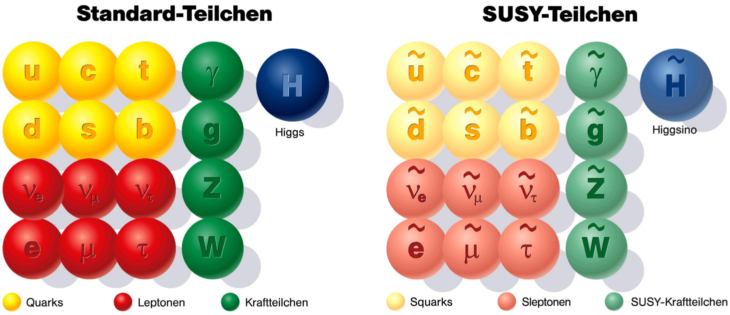
- defining the Pauli-Lubanski-Vector W_μ

$$W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}, \qquad X^\mu = Q\sigma^\mu\bar{Q}$$

$$Y := W^\mu - \frac{1}{4}X^\mu, \qquad [Y_\mu, Y_\nu] = im\epsilon^{\mu\nu\sigma}Y^\sigma, \qquad \left(\frac{Y}{m}\right) = y\,(y+1)$$

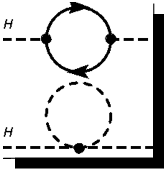


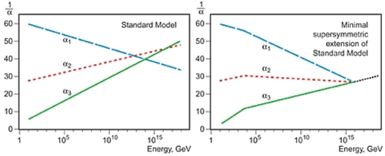
the particle spectrum



the consequences

- solution for the hierarchy-problem
- fewer divergencies
- local supersymmetry $\phi'_i(x) = U_i^j \phi_j(x) \rightarrow U_i^j(x)\,\phi_j(x)$
  $\rightarrow$ SUGRA (with ART in the low energy limit)
- TOE's only consistent with space-time supersymmetry
- quark confinement
- breaking electroweak interaction is a consequence of supersymmetry breaking





Wess-Zumino-Model '74

- take the action-functional

$$S[\phi] = \int d^4x \mathcal{L}(\phi, \partial\phi)$$

- calculate the invariance under variation of the Poincaré-Group $\mathcal{P}$

$$\delta S[\phi] = \int d^4x \delta\mathcal{L}$$

- with the extension

$$\left(x_1, x_2, x_3, x_4, \theta^1, \theta^2, \bar{\theta}_{\dot{1}}, \bar{\theta}_{\dot{2}}\right)$$

- and transformations

$$\theta \to \theta + \eta, \qquad \bar{\theta} \to \bar{\theta} + \bar{\eta} \qquad x^\mu \to x^\mu + a^\mu - i\eta\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\eta}$$

the continuums lagrangian

- equation of motion should be invariant under supersymmetric transformations

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \mathcal{L}}{\partial\left(\partial\phi/\partial x_\mu\right)}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

- the simplest supersymmetric $\mathcal{L}$agrangian for the chiral multiplet

$$\mathcal{L}_{chiral} = \mathcal{L}_{scalar} + \mathcal{L}_{fermion} = -\partial^\mu \phi^* \partial_\mu \phi - i\psi^t \bar{\sigma}^\mu \partial_\mu \psi$$

- the interaction between fermion- and boson-fields should be yukawa-like and renormalisable. the product $\phi^\dagger \phi$ leads to a vektor-superfield
- Super-Yang-Mills with symmetry breaking term

$$\mathcal{L} = \mathcal{L}_{SYM} + m\bar{\lambda}\lambda$$

on-shell action/Curci-Veneziano lattice action ('87)

- ...further constructions and restrictions...
- leads to the euclidean on-shell continuum-action

$$S_{SYM} = \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} \bar{\lambda}^a \gamma^\mu \nabla_\mu \lambda^a \right\}$$

- putting it on the lattice leads to $S_{lat} = S_g + S_f$ with

$$\begin{aligned} S_g[U] &= \beta \sum_x \sum_{\mu\nu} \left[1 - \frac{1}{N_c} \mathrm{ReTr} U_{\mu\nu} \right] \\ S_f\left[U, \bar{\lambda}, \lambda\right] &= \frac{1}{2} \sum_x \bar{\lambda}(x) \lambda(x) + \\ & \frac{\kappa}{2} \sum_x \sum_\mu \left[\bar{\lambda}(x+\hat{\mu}) V_\mu(x) (r + \gamma_\mu) \lambda(x) \right. \\ & \left. + \bar{\lambda}(x) V^T_\mu(x) (r - \gamma_\mu) \lambda(x+\hat{\mu}) \right] \end{aligned}$$

some comments on Q and V-matrix

- by defining the Q-Matrix

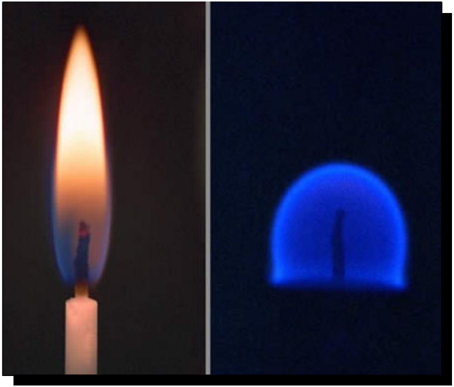
$$Q_{y,x}[U] \equiv \delta_{yx} - \kappa \sum_\mu \left[\delta_{y,x+\hat{\mu}} (1+\gamma_\mu) V_\mu(x) + \delta_{y+\hat{\mu}} (1-\gamma_\mu) V_\mu^T(y)\right]$$

- we can write S_f more compactly as

$$S_f = \frac{1}{2} \sum_{xy} \bar{\lambda}(x) Q_{x,y} \lambda(y) = \frac{1}{2} \sum_{xy} \lambda^T(x) \mathcal{C} Q_{x,y} \lambda(y)$$

- the adoint matrices have the form

$$[V_\mu(x)]_{ab} \equiv 2\mathrm{Tr}\left[U_\mu^\dagger(x) T^a U_\mu(x) T^b\right]$$



multi-bosonic representation

- it's not feasible to simulate Grassmann fields directly, because $e^{-S_F} = e^{-\bar{\phi} D\phi}$ is not positive $\rightarrow$ poor importance sampling
- we therefore integrate out the fermion fields to obtain the fermion determinant

$$\int [d\lambda]\, e^{-S_f} = \int [d\lambda]\, e^{-\frac{1}{2}\overline{\lambda} Q \lambda} = \pm\sqrt{\det Q}$$

- questions concerning the $\pm$-sign $\rightarrow$ ask Jaïr
- now we turn around - that thing is calles "bosonification", with $\sqrt{\det Q} = \left[\det\left(Q^\dagger Q\right)\right]^{\frac{1}{4}}$. In QCD you have

$$\det\left(Q^\dagger Q\right) = \int \left[d\phi^\dagger d\phi\right] \exp\left(-\sum_{xy} \phi_y^\dagger \left[Q^\dagger Q\right]^{-1}_{yx} \phi_x\right)$$

polynomial approximation

- use the approximation

$$\lim_{n\to\infty} P_n(x) = \left[\frac{1}{x}\right]^{\frac{1}{4}} \qquad \forall x \in [\epsilon, \lambda]$$

keep in mind the condition number

$$\sqrt{\lambda/\epsilon}$$

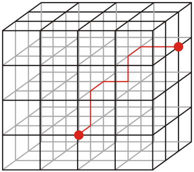
- choose the polynom

$$P\left(\tilde{Q}^2\right) = c_0 \left(\tilde{Q} - \rho_1\right)\left(\tilde{Q} - \rho_2\right)\ldots\left(\tilde{Q} - \rho_n\right)\left(\tilde{Q} - \rho_n^*\right)\ldots\left(\tilde{Q} - \rho_1^*\right)$$

- so in our simulation, we have

$$\sqrt{\det Q} = \left[\det\left(Q^\dagger Q\right)\right]^{\frac{1}{4}} = \int \left[d\phi^\dagger d\phi\right] \exp\left(-\sum_{xy} \phi_y^\dagger \left(P\left(\tilde{Q}^2\right)\right)_{yx} \phi_x\right)$$

the hamiltonian



- update the field globally
- takes large steps through configuration space
- we introduce a fictitious Hamiltonian

$$H[P, U, \phi] = \frac{1}{2} \sum_{x\mu j} P_{x\mu j}^2 + S_g[U] + S_f[U, \phi]$$

- the action plays the role of a fictitious potential
- HMC-Markov-Chain alternates two Markov-Steps:
  Molecular Dynamics Monte Carlo and
  Moment Refreshment (together they are ergodic)

equations of motion

- these are the hamilton equations of motion

$$\frac{dP_{x\mu j}}{d\tau} = -D_{x\mu j} S, \qquad \frac{dU_{x\mu}}{d\tau} = i P_{x\mu j} U_{x\mu}$$

- to update of momenta and fields

$$U'_{x\mu} = \exp\left\{\sum_j i2T_j P_{x\mu j} \Delta\tau\right\} U_{x\mu}, \qquad P'_{x\mu j} = P_{x\mu j} - D_{x\mu j} S\left[U,\phi\right]\Delta\tau$$

- you have to derive the fermionic derivative ($\left[D_{x\mu j} V_\mu\right]_{ab} = 2 f_{bjc}\left[V_\mu\right]_{ac}$)

$$D_{x\mu j} S_f\left[U,\phi\right] =$$
$$\sum_{k=0}^{n-1}\left(\phi_{1,a}^{(k)}(x)\left(D_{x\mu j}\tilde{Q}\right)\phi_{2,b}^{(k)\dagger}(y)\right) + \sum_{k=0}^{n-1}\left(\phi_{2,a}^{(k)}(x)\left(D_{x\mu j}\tilde{Q}\right)\phi_{1,b}^{(k)\dagger}(y)\right)$$

integrators

- Leapfrog integrator

$$T_{tot}\left(\Delta\tau\right) = T_P\left(\frac{\Delta\tau}{2}\right) T_U\left(\Delta\tau\right) T_P\left(\frac{\Delta\tau}{2}\right)$$

- Sexton-Weingarten integrator

$$T_{ges}\left(\Delta\tau\right) = T_U\left(\frac{\Delta\tau}{6}\right) T_P\left(\frac{\Delta\tau}{2}\right) T_U\left(\frac{2\Delta\tau}{3}\right) T_P\left(\frac{\Delta\tau}{2}\right) T_U\left(\frac{\Delta\tau}{6}\right)$$

- higher order Leapfrog integrator with multiple timescales

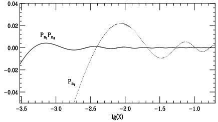
$$T_i\left(\Delta\tau_i\right) = T_{S_i}\left(\frac{\Delta\tau_i}{2}\right) \left\{T_{i-1}\left(\Delta\tau_{i-1}\right)\right\}^{N_i} T_{S_i}\left(\frac{\Delta\tau_i}{2}\right)$$

- higher order Sexton-Weingarten integrator with multiple timescales

$$T_i\left(\Delta\tau_i\right) = T_{S_i}\left(\frac{\Delta\tau_i}{6}\right) \left\{T_{i-1}\left(\frac{\Delta\tau_{i-1}}{2}\right)\right\}^{N_{i-1}} T_{S_i}\left(\frac{2\Delta\tau_i}{3}\right) \left\{T_{i-1}\left(\frac{\Delta\tau_{i-1}}{2}\right)\right\}^{N_{i-1}} T_{S_i}\left(\frac{\Delta\tau_i}{6}\right)$$

speed improvements

- two-step polynomial $\frac{1}{x} \equiv P_{n_1,n_2}(x) = P'_{n_1}(x)\, P''_{n_2}(x)$ with noisy correction

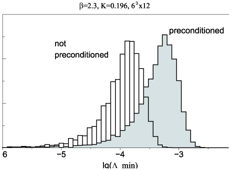


$$\frac{\mathrm{e}^{\eta^\dagger P''_{n_2}(\tilde{Q})\eta}}{\int \mathcal{D}[\eta]\, \mathrm{e}^{\eta^\dagger P''_{n_2}(\tilde{Q})\eta}}$$

- even-odd preconditioning ($\tilde{Q} = Q\gamma_5$)

$$\tilde{Q} = \begin{pmatrix} \gamma_5 & -\gamma_5 \kappa M_{even-odd} \\ -\gamma_5 \kappa M_{odd-even} & \gamma_5 \end{pmatrix}$$

$$\rightarrow \det \tilde{Q} = \det\left(\mathbb{1} - \kappa^2 M_{oe} M_{eo}\right)$$



- determinant breakup $\det \tilde{Q}^2 = \left\{ \left(\det \tilde{Q}^2\right)^{\frac{1}{n_B}} \right\}^{n_B}$

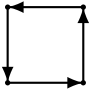
gauge action improvement

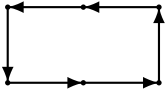
- both terms can be optimized

$$S = \underset{\text{DBW2}}{\underset{\downarrow}{S_g}} + \underset{\text{STOUT}}{\underset{\downarrow}{S_f}}$$

- a possible gauge action is

$$S = \beta_{11} \sum_{plaq} \mathrm{ReTr}\left(1 - \frac{1}{3} U_{plaq}\right) + \beta_{12} \sum_{plaq} \mathrm{ReTr}\left(1 - \frac{1}{3} U_{rect}\right)$$

$U_{plaq} =$ 

$U_{rect} =$ 

Wilson	TISym	Iwasaki	DBW2
$\beta_{12} = 0$	$\beta_{12} = -1/12$	$\beta_{12} = -0.091$	$\beta_{12} = -1.4088$

STOUT link smearing

- the $(n+1)^{th}$ stout smeared "thick" link obtained iteratively from the n^{th} level

$$U_{\mu}^{(n+1)}(x) = \mathrm{e}^{iQ_{\mu}^{(n)}} U_{\mu}^{(n)}(x)$$

- with

$$Q_{\mu}(x) = \frac{i}{2}\left(\Omega_{\mu}^{\dagger}(x) - \Omega_{\mu}(x)\right) - \frac{i}{2N}\mathrm{Tr}\left(\Omega_{\mu}^{\dagger}(x) - \Omega_{\mu}(x)\right)$$

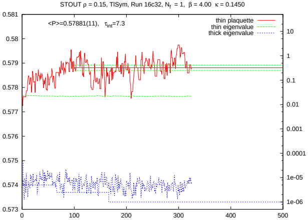
- and

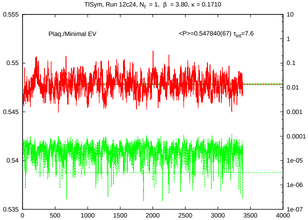
$$\Omega_{\mu}(x) = C_{\mu}(x)\,U_{\mu}^{\dagger}(x)$$

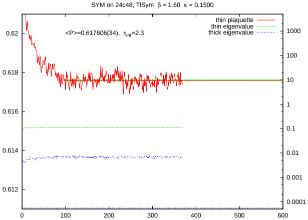
- the staples C_{μ} are defined as

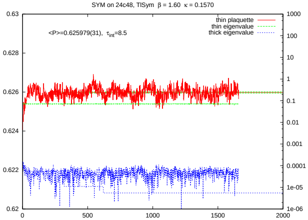
$$\begin{aligned} C_{\mu}(x) &= \sum_{\nu\neq\mu}\rho_{\mu\nu}\Big(U_{\nu}(x)\,U_{\mu}(x+\hat{\nu})\,U_{\nu}^{\dagger}(x+\hat{\mu}) \\ &\quad + U_{\nu}^{\dagger}(x-\hat{\nu})\,U_{\mu}(x-\hat{\nu})\,U_{\mu}(x-\hat{\nu})\,U_{\nu}(x-\hat{\nu}+\hat{\mu})\Big) \end{aligned}$$

some data from the analysis









the purpose of matrix inversions

- examine the Gluino-Propagator

$$\left\langle T\left\{\lambda\left(x\right)\bar{\lambda}\left(x\right)\right\}\right\rangle = \left\langle T\left\{\lambda\left(x\right)\lambda\left(x\right)\right\}\right\rangle \mathcal{C} = 2\left[\frac{\delta^2 \ln \mathcal{Z}\left[J\right]}{\delta J\left(x\right)\delta J\left(y\right)}\right]\mathcal{C}$$

- with the given partition function $\mathcal{Z}$

$$\mathcal{Z} = \int \mathcal{D}\left[\lambda\right] e^{-\frac{1}{2}\lambda\mathcal{C}Q\lambda}.$$

- we have to solve

$$\left\langle T\left\{\lambda\left(x\right)\bar{\lambda}\left(x\right)\right\}\right\rangle \quad = \quad \left\langle Q^{-1}\left[U\right]\right\rangle$$

- $\rightarrow$ inversion of sparse matrices

conjugate gradient algorithm I

- instead of solving $z = Q^{-1}\omega$ we solve

$$Qz = \omega$$

($\omega =$ a given source, $Q =$ the fermion matrix, $z =$ solution vector)

- to find z, we use **the conjugate gradient algorithm**.
- the basic idea of CG is, that equivalent to solve $Qz = \omega$ is extremising

$$E(z) := \langle \omega, z \rangle - 1/2 \langle Qz, z \rangle .$$

- the gradient of E at z_k is

$$g_k = \omega - Qz_k$$

- *conjugate* gradient means now, to minimize E in a direction p_k instead of g_k. This direction is Q-conjugated, which means

$$\langle Qp_i, p_j \rangle = 0$$

conjugate gradient algorithm II

- the algorithm step by step
  1. take a source ω and set initially $\omega = z_0$
  2. calculate the residuum
  $$p_0 = r_0 = \omega - Qz_0$$
  3. for $n = 1, 2, \ldots$
  $$a_n = \frac{|r_n|^2}{\langle p_n, Qp_n\rangle}, \qquad z_{n+1} = z_n + a_n p_n, \qquad r_{n+1} = r_n - a_n Q p_n$$
  4. if $|r_{n+1}|^2 < \delta$ then the solution is z_{n+1}, else calculate
  $$b_n = |r_{n+1}|^2 / |r_n|^2, \qquad p_{n+1} = r_{n+1} + b_n p_n$$
  and proceed with iteration
- is valid only for positiv definite hermitian matrices, $\rightarrow$ extend to $B^\dagger A$ it can be used for any hermitian matrix A

krylov spaces I

- consider a system of linear equations $Ax = b$ and the residual vector $r \equiv b - Ax_i$ for an approximate solution x_i
- rewriting the system as

$$(I - (I - A))\, x = b$$

- leads to basic iteration

$$\begin{aligned} x_i &= b + (I - A)\, x_{i-1} \\ &= x_{i-1} + r_{i-1} \\ &= x_{i-2} + r_{i-2} + r_{i-1} \\ &\vdots \\ &= x_0 + r_0 + r_1 + \ldots + r_{i-1}. \end{aligned}$$

krylov spaces II

- multiply $x_i = x_{i-1} + r_{i-1}$ with A from the left

$$Ax_i = Ax_{i-1} + Ar_{i-1}$$

- and substract from b

$$\begin{aligned} b - Ax_i &= b - Ax_{i-1} + Ar_{i-1} \\ r_i &= r_{i-1} - Ar_{i-1} \\ &= (I - A)\, r_{i-1} \end{aligned}$$

- so finally we get

$$\begin{aligned} x_i &= x_0 + r_0 + (I - A)\, r_0 + \ldots + (I - A)^{i-1}\, r_0 \\ &= x_0 + \left[r_0, Ar_0, A^2 r_0, \ldots, A^{i-1} r_0\right] \end{aligned}$$

krylov spaces III

- this linear space defines the krylov subspace

$$\mathcal{K}_m(A, r) = \mathrm{span}\{r, Ar, \ldots, A^{m-1}r\}.$$

- convergence is measured by the residual $r_n = |b - Ax_n|$.
- more specificially, we seek an approximate solution x_n in $\mathcal{K}_n$ by imposing the petrov-galerkin condition

$$r_n \equiv b - Ax_n \perp \mathcal{L}_n$$

  where $\mathcal{L}_n$ is an n-dimensional subspace
- two broad choices:
  - $\mathcal{L}_n = \mathcal{K}_n(A; r_0) \leftrightarrow$ orthognoalisation (Arnoldi, GMRES, CG, GCR...)
  - $\mathcal{L}_n = \mathcal{K}_n\left(A^\dagger; r_0\right) \leftrightarrow$ bi-orthognoalisation (Lanczos, BCG, BiCGstab...)

on which circumstances is matrix deflation feasible?

- stochastic estimator technique

$$\left\langle \eta_i^\dagger Z_i \right\rangle_{N_{est}} \overset{\text{N}_{\text{est}}\to\infty}{=} Q_{ii}^{-1}$$

- collect informations about Q in each CG for the next step
- feed CG with a

galerkin-projected vector

$$x_0 = W\left(W^T A W\right)^{-1} W^T b.$$

- $\longrightarrow$ convergence will raise

SET
$Qz_1 = \eta_{1\alpha}$
$\downarrow$
$Qz_2 = \eta_{2\alpha}$
$\downarrow$
$Qz_i = \eta_{i\alpha}$
$\vdots$
$Qz_N = \eta_{N\alpha}$

deflation: the stathopoulos-orginos algorithm

- InitCG

ALGORITHM 1: BASISITERATION

iterative solution of $Ay = c$

Initialisation

choose y_0;

$s_0 = c - Ay_0$;

$\omega_0 = s_0$;

Iteration

for $j = 0, 1, \ldots$ **until** covergence **do**

$\gamma_j = \left(s_j, s_j\right) / \left(\omega_j, A\omega_j\right)$;

$y_{j+1} = y_j + \gamma_j \omega_j$;

$s_{j+1} = s_j - \gamma_j A\omega_j$;

$\delta_{j+1} = \left(s_{j+1}, s_{j+1}\right) / \left(s_j, s_j\right)$;

$\omega_{j+1} = s_{j+1} + \delta_{j+1}\omega_j$;

end do

ALGORITHM 2: INITCG

iterative solution of $Ax = b$

Initialisation

choose x_{-1};

$r_{-1} = b - Ax_{-1}$;

$x_0 = x_{-1} + W\left(W^T AW\right)^{-1} W^T r_{-1}$;

$r_0 = b - Ax_0$;

$p_0 = r_0$;

Iteration

for $j = 0, 1, \ldots$ **until** covergence **do**

$\alpha_k = (r_k, r_k) / (p_k, Ap_k)$;

$x_{k+1} = x_k + \alpha_k p_k$;

$r_{k+1} = r_k - \alpha_k Ap_k$;

$\beta_{k+1} = \left(r_{k+1}, r_{k+1}\right) / (r_k, r_k)$;

$p_{k+1} = r_{k+1} + \beta_{k+1} p_k$;

end do

eigCG

- generate an initial V with restarting-CG
- $T_m = \left(W^T A W\right)^{-1}$ is the lanczos-matrix
- in 8. and 9. we use raileigh ritz, to compute an orthonormal ritz basis for space $[Y, \tilde{Y}]$
- return nev ritz vectors from V

1. $V = [\,]$;
2. **for** $j = 0, 1, \ldots$ **until** covergence **do**
3. standard CG iteration
4. update three elements of T_j
5. **if** (size $(V, 2) == m$)
6.   solve $T_m Y = Y M$, for nev lowest eigenpairs
7.   solve $T_{m-1} \tilde{Y} = \tilde{Y} \tilde{M}$, for nev lowest eigenpairs
8. $[Q, R] = qr\left(\left[Y, \tilde{Y}, 0\right]\right)$, and $H = Q^H T_m Q$
9.   solve $HZ = ZM$ for 2 nev lowest eigenpairs
10.   Restart: $V = V\,(QZ)$ and $T_{2nev} = M$
11.   set the $2nev + 1$ column of T_{2nev+1} as $V^H A r_j$
12. **endif**
13. $V = \left[V, r_j / \|r_j\|\right]$
14. **end CG**

solver convergence

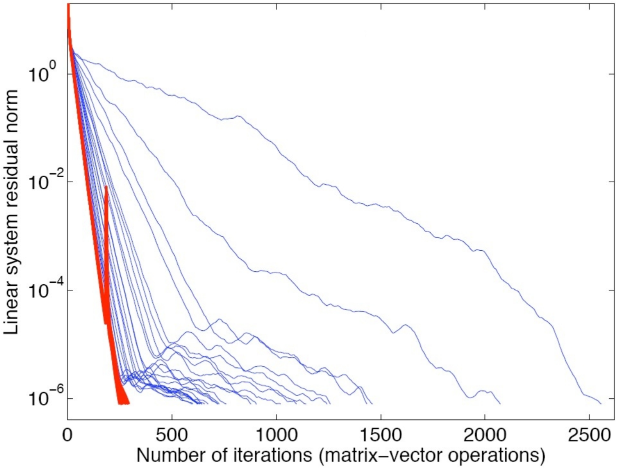


Figure: convergence of the solvers. the blue ones are the 24 incremental eigCG iterations, red the last 24 init-CG iterations

deflation: the lüscher algorithm

- at the beginning of each MD-trajectory, there will be fermion-fields $\phi_l(x), l = 1, \ldots, N_s$ stochastically gernerated through a so called *smoothing procedure*
- then, they will be projected on non-overlapping Blocks Λ with

$$\phi_l^\Lambda(x) = \begin{cases} \phi_l(x) & \text{wenn } x \in \Lambda, \\ 0 & \text{sonst} \end{cases}$$

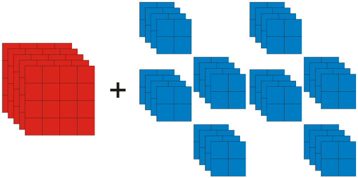


Figure: often used blocksize 4^4

mode projection I

- a given field ψ can be projected with an orthogonal projector P on the space $\mathcal{S}$, which is spanned by the orthonormalbasis $\phi_1(x), \ldots, \phi_N(x)$

$$P\psi(x) = \sum_{k=1}^{N} \phi_k(x)\,(\phi_k, \psi)\,.$$

- the complete system is combined by the "inner" system $\mathcal{S}$ and an "outer" complementary System $\mathcal{S}^{\perp}$

$$\psi(x) = \chi(x) + \sum_{k,l=1}^{N} \phi_k(x)\left(A^{-1}\right)_{kl}(\phi_l, \eta)$$

- here we used the so called *little Dirac-Operator*

$$A_{kl} = (\phi_k, D\phi_l)\,, \qquad k,l = 1,\ldots,N.$$

mode projection II

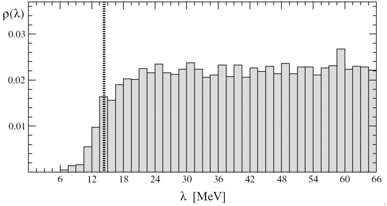


Figure: deviation of the smallest eigenvalues

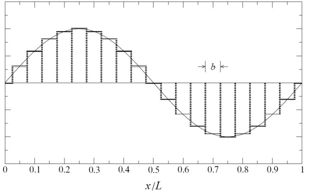


Figure: approximation of the lowest modes by a constant

comparison of the various methods

	Morgan/Wilcox	**Stath./Orginos**	**Lüscher**
Solver	GMRES / BiCGStab	CG	GCR
Matrix Type	non-herm. (Algebraic)	herm. (Algebraic)	non-herm. (Lattice)
Simultaneous solve	yes	yes	no
Eigenvalue use for multiple rhs's	every cycle (GMRES) beginning (BiCGStab)	every cycle, beginning ($s \leq s_1$) restart ($s > s_1$)	every outer iteration
Algorithm acceleration	mild	large	small

Table: some points of comparison for the three algorithms considered

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